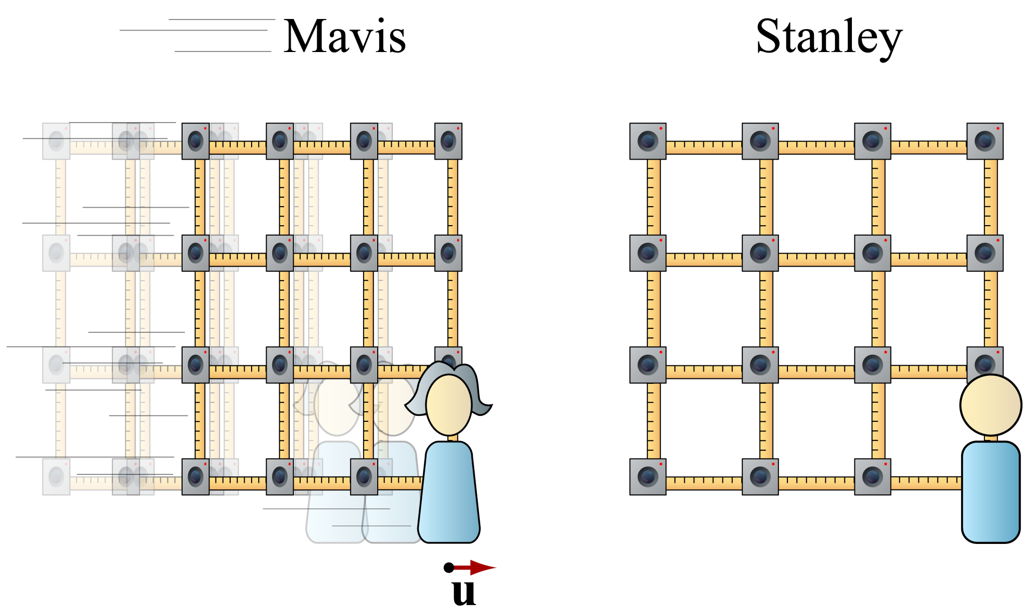
# Special relativity




The convention of naming frames "Mavis" and "Stanley" is borrowed from Young and Freedman's *University Physics* 10th ed.

## Definitions and postulates

**observer** or **frame of reference** – batch of timestamped, local video recordings obtained from a fleet of security cameras that have been **attached** to various locations on a rigid cage built from rigid metersticks (co-moving), and that have been **"synchronized"** using a light pulse

**event** – a particular position and time

**kinematics** – When describing the video recordings obtained by any particular frame of reference, changes and rates are defined in the classical way for pairs of events.

$$\Delta x, \quad \Delta t, \quad v_{x,\text{AVG}} := \frac{\Delta x}{\Delta t}, \quad a_{x,\text{AVG}} := \frac{\Delta v_x}{\Delta t}$$

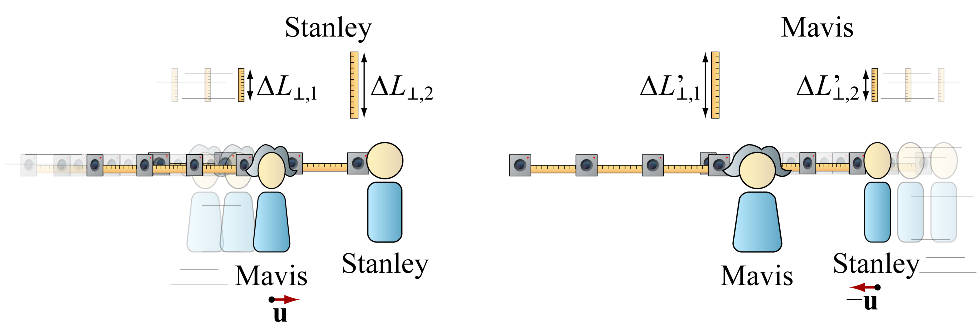
**Postulate 1:** All inertial observers agree on the same laws of physics.

**Postulate 2:** All observers measure that the speed of light is $c = 3.00 \times 10^8\ \frac{\text{m}}{\text{s}}$.

## Results

### Transverse lengths are agreed on

Suppose the transverse length interval between two ends of an object measured using a frame of reference, say, decreased the faster the object moved past that frame of reference.




Then, by sending two parallel rulers toward each other and attaching one frame of reference to each ruler, it would be possible to obtain a contradictory set of described events, so $\Delta L_{\perp} = \Delta L_{\perp}$.

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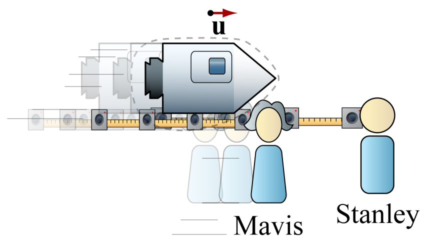
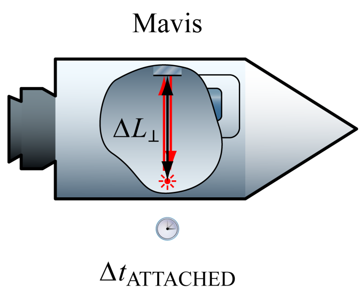
### More definitions

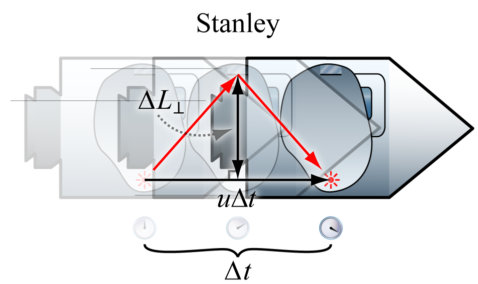
**proper time** $\Delta t_{\mathrm{ATTACHED}}$ – time interval between two events locally reported on the clockface of a single clock that is part of a frame of reference in which the two events occur at the same position

**proper length** $\Delta L_{\mathrm{ATTACHED}}$ – length interval between ends of an object as measured by the meter sticks of a frame of reference in which the object is stationary

### Time dilation




| Mavis | Stanley |
|---|---|
| $2\Delta L_\perp = c\Delta t_{\mathrm{ATTACHED}}$ <br> $\frac{2\Delta L_\perp}{c} = \Delta t_{\mathrm{ATTACHED}}$ | $2\sqrt{\Delta L_\perp^2 + \left(\frac{u\Delta t}{2}\right)^2} = c\Delta t$ <br> $\frac{4}{c^2}\left(\Delta L_\perp^2 + \left(\frac{u\Delta t}{2}\right)^2\right) = \Delta t^2$ <br> $\left(\frac{2\Delta L_\perp}{c}\right)^2 + \left(\frac{u\Delta t}{c}\right)^2 = \Delta t^2$ |

$$\Delta t_{\mathrm{ATTACHED}}^2 + \left(\frac{u\Delta t}{c}\right)^2 = \Delta t^2$$

$$\Delta t = \frac{\Delta t_{\mathrm{ATTACHED}}}{\sqrt{1 - \left(\frac{u}{c}\right)^2}}$$

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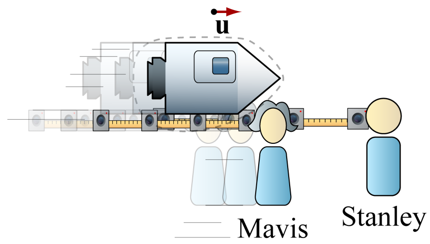
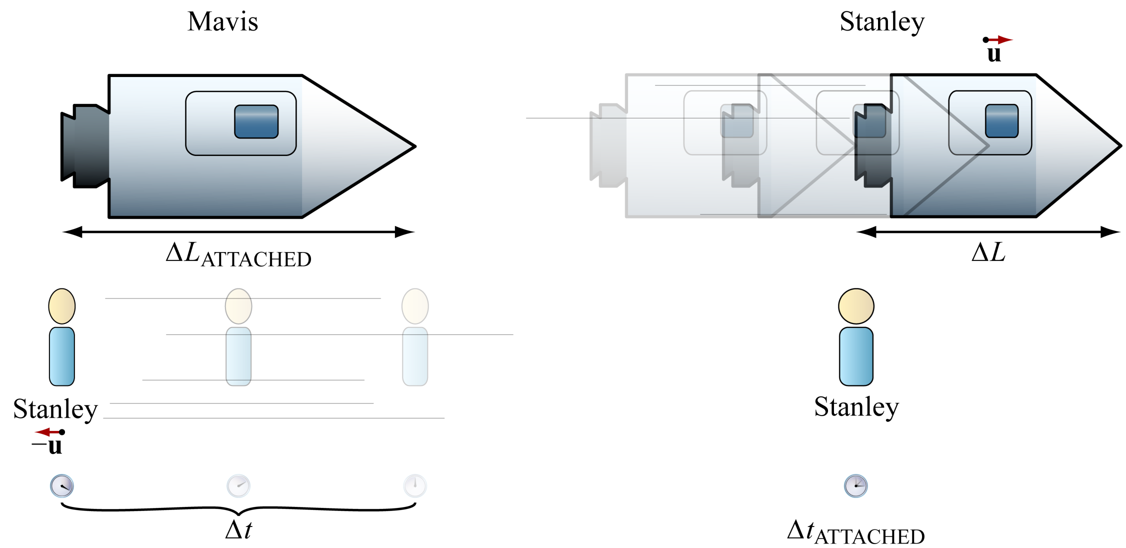
## Length contraction




| Mavis | Stanley |
|---|---|
| $\Delta L_{\text{ATTACHED}} = u\Delta t$ <br> $u = \dfrac{\Delta L_{\text{ATTACHED}}}{\Delta t}$ | $\Delta L = u\Delta t_{\text{ATTACHED}}$ <br> $u = \dfrac{\Delta L}{\Delta t_{\text{ATTACHED}}}$ |

$$\Delta L = \Delta L_{\text{ATTACHED}} \sqrt{1 - \left(\frac{u}{c}\right)^2}$$

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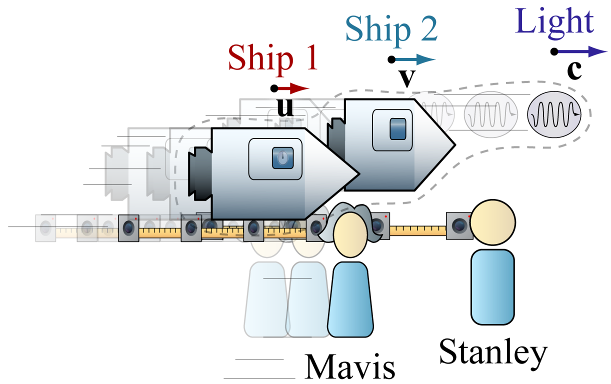
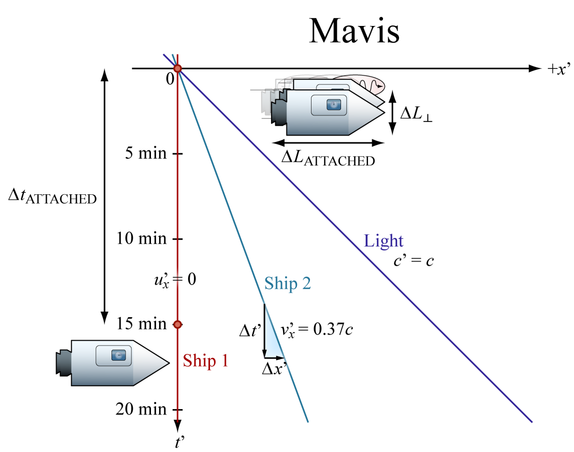
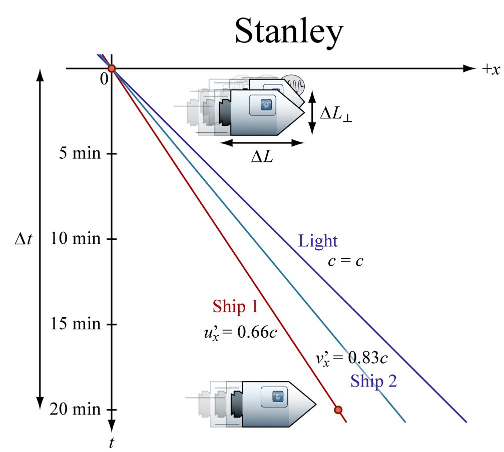
## Kinematics




$$\Delta t = \frac{\Delta t_{\text{ATTACHED}}}{\sqrt{1 - \left(\frac{u}{c}\right)^2}}$$

$$v'_x = \frac{v_x - u_x}{1 - \frac{u_x v_x}{c^2}}$$

$$\Delta L = \Delta L_{\text{ATTACHED}} \sqrt{1 - \left(\frac{u}{c}\right)^2}$$

$$f_{\text{OBS}} = f_{\text{SRC}} \sqrt{\frac{1 + \frac{v_{\text{APPROACH}}}{c}}{1 - \frac{v_{\text{APPROACH}}}{c}}}$$

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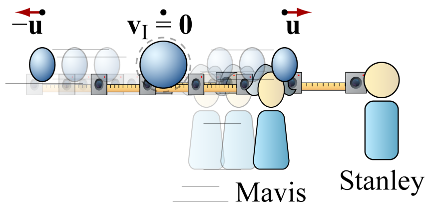
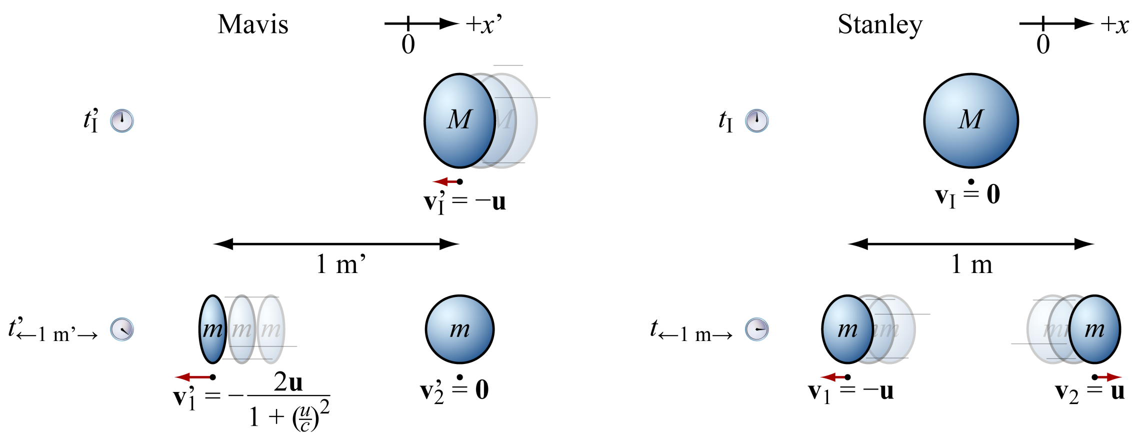
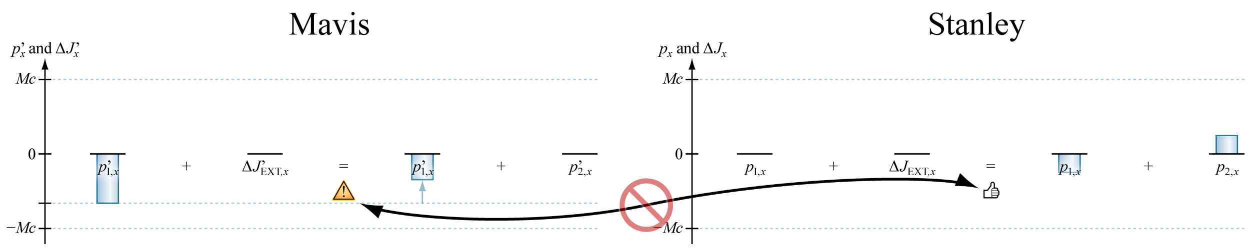
## Non-relativistic mechanics breaks down when relative speeds are comparable to $c$



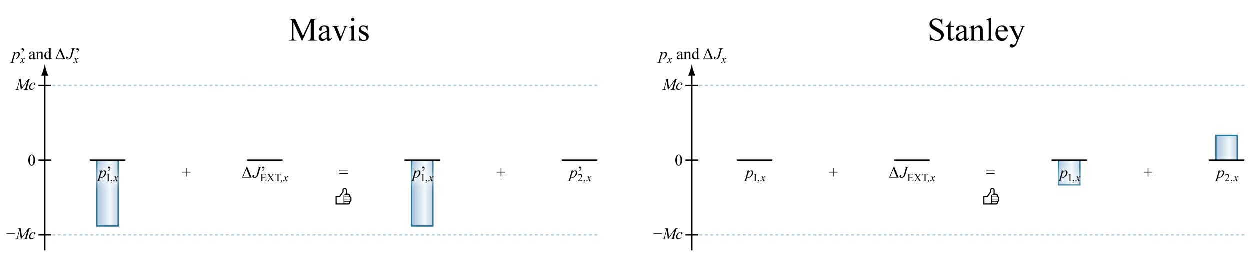

## Non-relativistic impulse-momentum formulation




$$p_x := mv_x$$

$$\Sigma\vec{\mathbf{p}}_i + \overbrace{\sum_{\substack{\text{EXT}\\\text{ON SYS}}} \Delta\vec{\mathbf{J}}_{\text{F}}}^{\Delta\vec{\mathbf{J}}_{\Sigma\vec{\mathbf{F}},\text{EXT ON SYS}}} = \Sigma\vec{\mathbf{p}}_f$$

## Relativistic impulse-momentum formulation




$$p_{\text{MASSIVE},x} := \frac{mv_x}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$p_{\text{PHOTON}} = \frac{hf}{c}$$

$$\Sigma\vec{\mathbf{p}}_i + \overbrace{\sum_{\substack{\text{EXT}\\\text{ON SYS}}} \Delta\vec{\mathbf{J}}_{\text{F}}}^{\Delta\vec{\mathbf{J}}_{\Sigma\vec{\mathbf{F}},\text{EXT ON SYS}}} = \Sigma\vec{\mathbf{p}}_f$$

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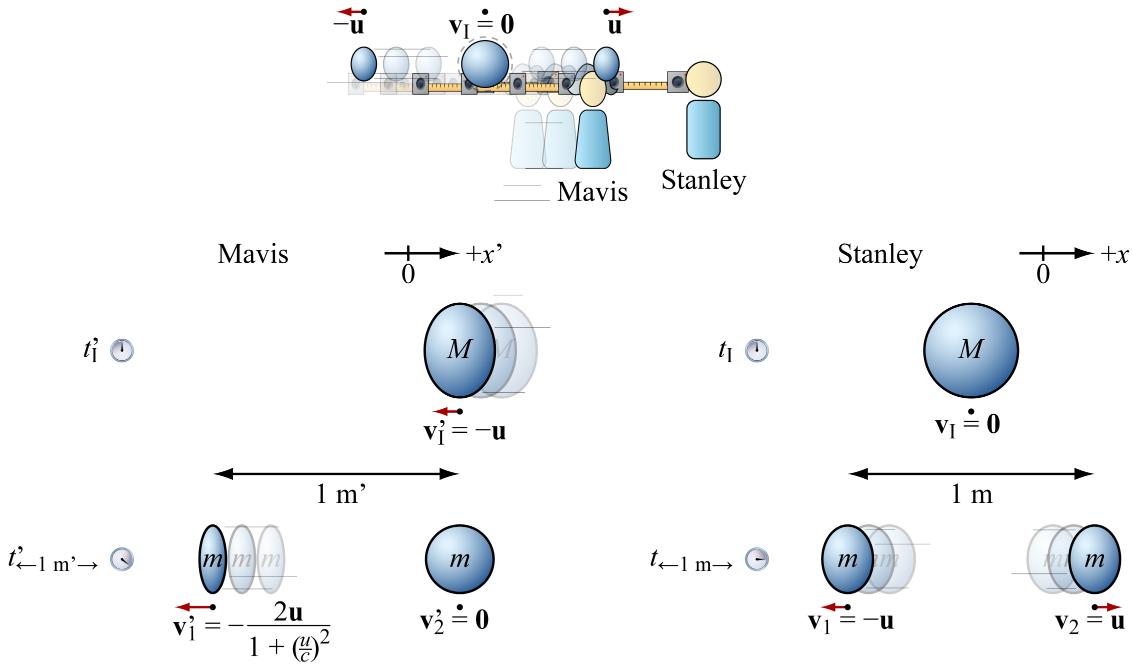
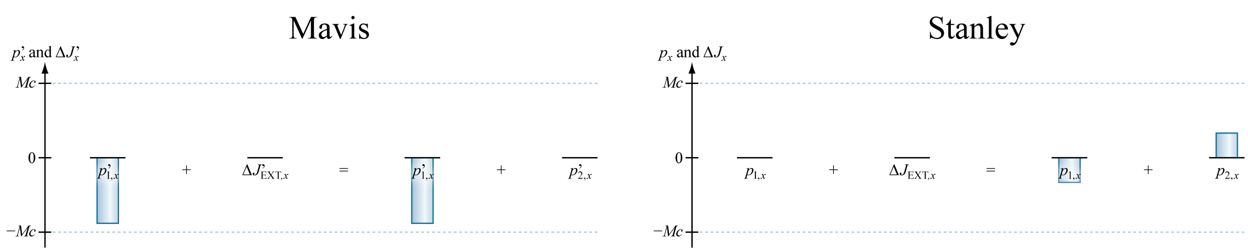
## Relativistic mechanics




## Relativistic impulse-momentum formulation

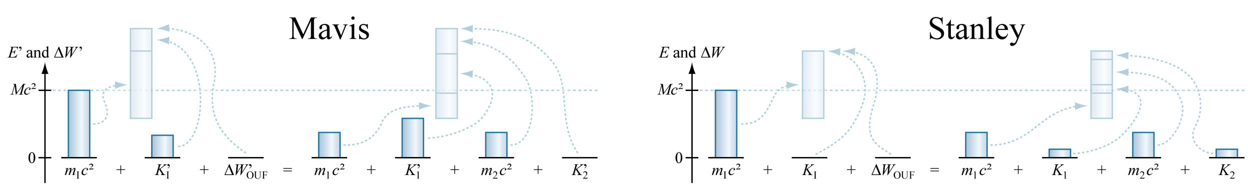



$$p_{\mathrm{MASSIVE},x} := \frac{m v_x}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$p_{\mathrm{PHOTON}} = \frac{hf}{c}$$

$$\Sigma\vec{\mathbf{p}}_i + \overbrace{\sum_{\substack{\mathrm{EXT}\\ \mathrm{ON\ SYS}}} \Delta\vec{\mathbf{J}}_{\mathrm{F}}}^{\Delta\vec{\mathbf{J}}_{\Sigma\vec{\mathbf{F}},\mathrm{EXT\ ON\ SYS}}} = \Sigma\vec{\mathbf{p}}_f$$

## Relativistic work-energy formulation




$$E_{\mathrm{MASSIVE}} := \frac{mc^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$= mc^2 + K$$

$$E_{\mathrm{PHOTON}} = hf$$

$$\Sigma m_i c^2 + \Sigma K_i + \Sigma h f_i + \Sigma \Delta W_{\mathrm{OUF}} = \Sigma m_f c^2 + \Sigma K_f + \Sigma h f_f$$